Lecture Seventeen: Theory of Relativity

17.1 Collapse of a Massive Star to form a Black Hole

Do black holes exist in Nature? A star like the Sun is held up by radiation pressure. It is burning hydrogen to make helium releasing about 26 Mev for each atom of He that is formed. What happens when all of the nuclear fuel is used up? The star must collapse to a high density. In fact, we expect that the Sun will collapse to a form a white dwarf – a star with a radius of about 5000 km and a spectacularly high mean density of about 10^6 g cm^{-3}.

Astronomers knew about white dwarfs a long time ago. For example, in 1915 Adams discovered that the companion of the bright star Sirius had a 'white' spectrum from which he deduced an effective temperature of 8000 K. The mass of the companion (known as Sirius B) was estimated to be about $0.8M_\odot$ from observations of the binary orbit. Given its observed flux and temperature, the blackbody radiation law could be used to infer a radius of $\sim 20,000$ km and so, given its mass, there could be no escape from the conclusion that Sirius B was a very compact star of extraordinarily high density. In 1925, Adams showed further that the spectral lines of Sirius B were gravitationally redshifted – as expected from General Relativity. Here is what Eddington said about this discovery in his classic book *The Internal Constitution of the Stars* written in 1926:

> "Prof. Adams has killed two birds with one stone; he has carried out a new test of Einstein's general theory of relativity and he has confirmed our suspicion that matter 2000 times denser than platinum is not only possible, but is present in the Universe."

This, incidentally, is a good example of why astronomy is so interesting. Astronomy turns up strange objects in which conditions are much more extreme than anything we encounter here on Earth. This allows us to study physics in unusual regimes, where we can hope to learn something fundamentally new . Examples include neutron stars, quasars, relativistic jets, accretion discs, gamma ray bursts and cosmological fluctuations in the early Universe.

Nobody knew how to explain white dwarfs. What holds them up? What provides the pressure? The answer had to await the development of quantum mechanics and the formulation of Fermi-Dirac statistics. Fowler realised in 1926 that white dwarfs were held up by electron degeneracy pressure. The electrons in a white dwarf behave like the free electrons in a metal. Because of the Pauli exclusion principle, the electrons completely fill phase space up to a characteristic Fermi-energy. It is the Pauli exclusion principle that holds up a white dwarf. Chandrasekhar, in 1930, was on his way to Cambridge to start work as a research student under Eddington's supervision. In transit, Chandrasekhar realized that the more massive a white dwarf, the denser it must be and so the stronger the gravitational field. For white dwarfs about a critical mass of $1.4M_\odot$ (now called the Chandrasekhar limit), gravity would overwhelm degeneracy pressure and no stable solution was possible. The white dwarf would collapse to a point. Eddington was not impressed (though Chandrasekhar later won the Nobel prize for this work). Here is another quote from Eddington,

> “The star apparently has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few kilometres radius when gravity becomes strong enough to hold the radiation and the star at last can find peace.... I think that there should be a law of Nature to prevent the star from behaving in this absurd way.”

After the discovery of the neutron, people realized that at extremely high densities the electrons would react with the protons to form neutrons. A new stable configuration for a star was possible – neutron stars. A neutron star of one solar mass would have a radius of only 10 km. But as with white dwarfs, neutron stars have a maximum mass above which no stable configuration is possible. This maximum mass is believed to be about $3M_\odot$ or less – the exact value is uncertain because of uncertainties in the equation of state of matter at such high densities. Nevertheless, we now believe that Eddington was wrong – there is no law of Nature to prevent the gravitational collapse of a massive star. We believe that massive stars collapse to form *black holes*.

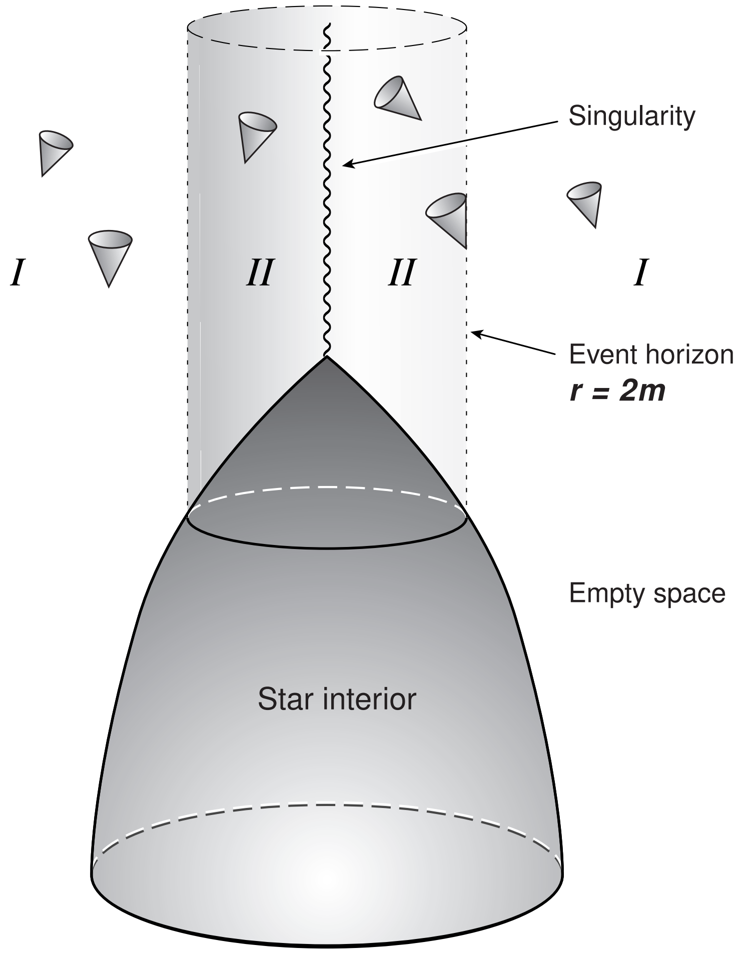

Figure 1: Collapse of a star to form a black hole.

Figure 1 shows a space time diagram of a collapsing star. Eventually, the radius of the star collapses to less than the Schwarzchild radius, at which point the formation of a black hole becomes inevitable.

Some theorists were very skeptical about the formation of black holes. The Schwarzchild solution is very special – it is exactly spherically symmetric by construction. In reality, a star will not be perfectly symmetric and so perhaps, as it collapses, the assymmetries amplify and avoid the formation of an event horizon. Perhaps black holes never form or are exceedingly rare in Nature. Penrose in the early 1960's applied global geometrical techniques to prove a famous series of 'singularity theorems'. These show that in realistic situations an event horizon (a closed trapped surface) will be formed and that there must exist a singularity within this surface, *i.e.* a point at which the curvature diverges and General Relativity ceases to be valid. I will touch on Penrose's approach in a later lecture. The singularity theorems were important in convincing people that black holes must form in Nature. In the rest of this section, I will look at some of the observational evidence for the existence of black holes. As we will see, there is compelling evidence that black holes do indeed exist. Furthermore, it should become possible within the next few years not only to measure the masses of black holes, but also to measure their angular momenta using powerful X-ray telescopes! Direct experimental probes of the strong gravity regime are now possible.

17.2 Compact binary systems

Although radiation cannot escape from black holes, one of the best way of finding candidate black holes is to search for luminous compact X-ray sources. The reason for this is that if a black hole has a stellar companion, the intense tidal field can pull gas from the companion producing an accretion disc around the black hole. A schematic picture is shown below. I will show you that accretion discs can radiate very efficiently.

Figure 2: Schematic picture of a compact binary system.

The following table summarizes the common classes of compact binaries. The compact object can be a white dwarf, neutron star, or a black hole.

Compact accreting binary systems			
	Compact object		
Companion Star	White Dwarf	Neutron Star	Black Hole
Early type, massive	None known	Massive X-ray binaries	Cyg X-I, LMC X-3
Late-type, low mass	Cataclysmic variables (e.g., Dwarf novae)	Low mass X-ray binaries	A0620-00

If you find a compact binary system, then you can set limits on the mass of the compact object from the dynamics of the binary orbit. If you find evidence for a compact object that is more massive than the Chandrasekhar limit, then you have good evidence that the object might be a black hole. In fact it is not so straightforward. What observers actually measure is the *mass function*

$$f(M) = \frac{PK^3}{2\pi G},$$

where P is the orbital period, and K is the radial velocity amplitude. For example, for the low mass X-ray binary A0620-00, the period is $P = 7.7$ hours and $K = 457$ km s^{-1}. From Kepler's laws we can show that the mass function is related to the masses of the compact object (M_1), the companion star (M_2) and the inclination angle i of the orbit to the plane of the sky by

$$f(M) = \frac{M_1^3 \sin^3 i}{(M_1 + M_2)^2}.$$

You can see from this equation that the mass function is a strict lower limit on the mass of the compact object. It is equal to the mass, $f = M_1$, *only* if $M_2 = 0$ and the orbit is viewed edge on ($\sin i = 1$). For example, for A0620-00, the lower limit on the mass of the compact object is $2.9 M_\odot$, and this makes it a very good black hole candidate because this mass limit is very close to the theoretical *upper* limit for the mass of a neutron star. In fact, it is possible to make reasonable estimates[1] for M_2 and $\sin i$ in this system leading to a probable mass of $\approx 10 M_\odot$ for the compact object – well into the black hole regime.

The following table from a review by Phil Charles summarizes the dynamical mass limits on some good black hole candidates (so called short X-ray transients). As you can see in several systems, V404 Cyg, G2000+25 and N Oph 77, the minimum mass inferred from the mass function is *well above* the theoretical maximum mass limit for a neutron star. As we understand things at present there can be no other explanation other than that the compact objects are black holes.

[1] An estimate of the mass M_2 can be made by measuring the spectral type and luminosity of the companion star. The inclination angle can be estimated from the shape of the stars light curve – searching for evidence of eclipsing by the compact object.

Table 1. Derived Parameters and Dynamical Mass Measurements of SXTs

Source	$f(M)$ $(M_\odot)$	ρ (g cm^{-3})	q $(= M_1/M_2)$	i	M_1 $(M_\odot)$	M_2 $(M_\odot)$	Ref.
V404 Cyg	6.08 ± 0.06	0.005	17 ± 1	55 ± 4	12 ± 2	0.6	[1-2]
G2000+25	5.01 ± 0.12	1.6	24 ± 10	56 ± 15	10 ± 4	0.5	[3-5]
N Oph 77	4.86 ± 0.13	0.7	> 19	60 ± 10	6 ± 2	0.3	[6-9]
N Mus 91	3.01 ± 0.15	1.0	8 ± 2	54^{+20}_{-15}	6^{+5}_{-2}	0.8	[13-15]
A0620-00	2.91 ± 0.08	1.8	15 ± 1	37 ± 5	10 ± 5	0.6	[16-18]
J0422+32	1.21 ± 0.06	4.2	> 12	$20 - 40$	10 ± 5	0.3	[19-20]
J1655-40	3.24 ± 0.14	0.03	3.6 ± 0.9	67 ± 3	6.9 ± 1	2.1	[10-12]
4U1543-47	0.22 ± 0.02	0.2	–	$20 - 40$	5.0 ± 2.5	2.5	[21]
Cen X-4	0.21 ± 0.08	0.5	5 ± 1	43 ± 11	1.3 ± 0.6	0.4	[22-23]

References

[1] Casares & Charles 1994; [2] Shahbaz et. al. 1994b; [3] Filippenko et. al. 1995a; [4] Beekman et. al. 1996; [5] Harlaftis et. al. 1996; [6] Filippenko et. al. 1997; [7] Remillard et. al. 1996; [8] Martin et. al. 1995; [9] Harlaftis et. al. 1997; [10] Orosz & Bailyn 1997; [11, 12] van der Hooft 1997, 1998; [13] Orosz et. al. 1996; [14] Casares et. al. 1997; [15] Shahbaz et. al. 1997; [16] Orosz et. al. 1994; [17] Marsh et al 1994; [18] Shahbaz et. al. 1994a; [19] Filippenko et. al. 1995b; [20] Beekman et. al. 1997; [21] Orosz et. al. 1998; [22] McClintock & Remillard 1990; [23] Shahbaz et. al. 1993.