Lecture Twenty One: Theory of Relativity

21.1 Solutions of the Friedmann equations

For simplicity, let us assume that the Universe is composed of an ideal fluid with zero pressure, $p = 0$. This is a good approximation for a non–relativistic gas or a "dust" of non-interacting particles. The equation of energy conservation in this case simply expresses mass conservation,

$$\frac{d(\rho R^3)}{dR} = 0 \ , \qquad i.e. \qquad \rho \propto \frac{1}{R^3}.$$

This solution tells us that the mass in a cubical box of proper length L is conserved as the box length L expands or contracts ($L \propto R(t)$),

$$M = \rho L^3 \propto \rho R^3 = \text{constant}.$$

Let us first investigate solutions of the Friedmann equations assuming that the cosmological constant Λ is zero. The scale factor $R(t)$ obeys the first order differential equation

$$R^2 + K = \frac{8\pi G}{3}\rho R^2, \tag{1}$$

and we have already established that mass conservation requires that the density varies as $\rho \propto R^{-3}$. We can fix the constant K in equation (1) by evaluating $\dot{R}$, R and ρ at the present time t_0.

$$\frac{K}{R_0^2} = \frac{8\pi G}{3}\rho_0 - \left(\frac{\dot{R}_0}{R_0}\right)^2. \tag{2}$$

The subscript 0 on, for example, R_0, denotes the value of the quantity at the present time ($R_0 \equiv R(t_0)$, $\rho_0 \equiv \rho(t_0)$, *etc*). Writing $H_0 = \dot{R}_0/R_0$ (we will see later that this is the Hubble constant), equation (2) becomes

$$\dot{R}^2 = \frac{8\pi G}{3}\rho_0\frac{R_0^3}{R} - \left(\frac{8\pi G}{3}\rho_0 - H_0^2\right)R_0^2,$$

which we can write as

$$\dot{R}^2 = \frac{8\pi G}{3}\rho_0\frac{R_0^3}{R} - \frac{8\pi G R_0^2}{3}\left[\rho_0 - \frac{3H_0^2}{8\pi G}\right]. \tag{3}$$

We don't actually have to solve this equation to see what the solutions will look like. It is easy to see that the evolution of $R(t)$ depends *on the sign of the term in square brackets.* Let us write equation (3) as

$$\dot{R}^2 = \frac{A}{R} - C, \qquad \text{where } A \text{ and } C \text{ are constants.} \tag{4}$$

Clearly, the constant A must be positive at all times, but the sign of the constant C depends on the sign of the term in square brackets in equation (3). It may be positive or negative depending on the value of the mean density of the Universe and the parameter H_0.

From equation (4), we can see that as $R \longrightarrow 0$, the term $A/R \longrightarrow \infty$ and so $\dot{R} \longrightarrow \infty$. The value of C is unimportant when R is small. Since R measures the scale of the Universe, we conclude that at early times the Universe must have started off from nearly infinite density with a very rapid expansion rate. This is the *Big Bang.* The early evolution does not depend on the value of the constant C, but the future evolution of the Universe depends critically on the value of C. From equation (4) we can distinguish three possible histories depending on the value of C:

$$\begin{cases} C = 0, & \dot{R} \longrightarrow 0 \ \text{ as } R \longrightarrow \infty, \\ C < 0, & \dot{R} \longrightarrow \text{const.} \ \text{ as } R \longrightarrow \infty, \\ C > 0, & \dot{R} = 0 \text{ at a maximum radius } R_m = \frac{A}{C}. \end{cases}$$

The future evolution therefore depends on the mean density of the Universe. $C = 0$ when the Universe has the *critical density*

$$\rho_c = \frac{3H_0^2}{8\pi G}. \tag{5}$$

Of course, the actual density of the Universe may be greater than or less than the critical density and so it is useful to define a *density parameter*,

$$\Omega = \frac{\rho}{\rho_c}, \tag{6}$$

which is just the ratio of the mean density of the Universe to the critical density. Evidently, $\Omega = 1$ if the density of matter in the Universe is exactly equal to the critical density. Notice from equation (2) that the curvature K of the Universe can be related to the value of the density parameter at some reference time (say, at the present time, Ω_0),

$$K = \frac{8\pi G R_0^2}{3}\left[\rho_0 - \rho_c\right],$$

i.e.,

$$\frac{K}{R_0^2} = (\Omega_0 - 1)H_0^2. \tag{7}$$

This is an important equation because it links the *geometry* of the Universe (via the curvature K) to the *density* of the Universe.

If $\Omega_0 < 1$, $K < 0$, and the Universe has negative curvature, (an open universe).

If $\Omega_0 = 1$, $K = 0$, and the Universe is spatially flat, (Einstein–de Sitter universe).

If $\Omega_0 > 1$, $K > 0$, and the Universe has positive curvature, (closed universe).

The solutions for $R(t)$ for these three cases are sketched in Figure 1.

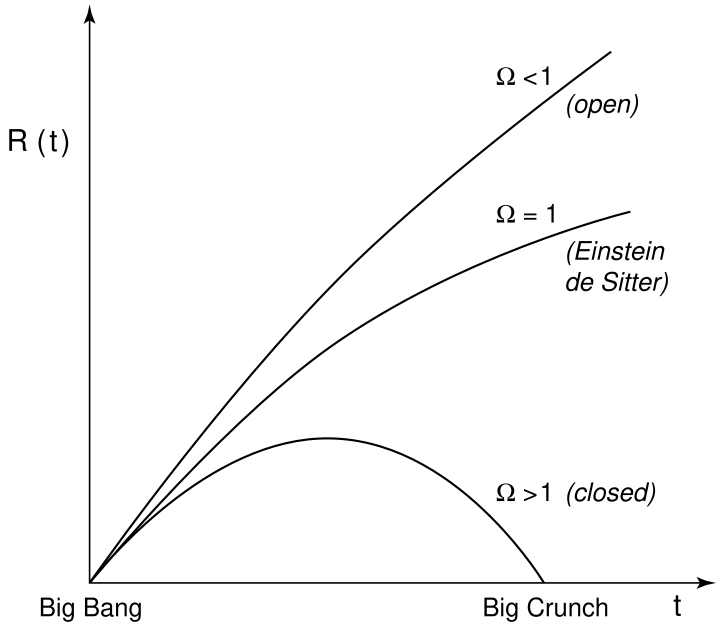

Figure 1: Schematic evolution of the scale factor in closed, open and Einstein de-Sitter universes.

If the Universe has exactly the critical density, then space is flat and the Universe is infinite. A critical density universe will expand forever, but with $\dot{R}$ tending to zero as R tends to infinity. If the mean density of the Universe exceeds the critical density, space has a finite volume and is positively curved (*i.e.* it is spatially closed). In a closed universe, the gravity of the matter is sufficiently strong that it can reverse the expansion. Such a universe will therefore expand to a maximum radius and then collapse, ending at a singularity. If the mean density of the Universe is lower than the critical density, space is infinite and negatively curved (*i.e.* it is spatially open). In this case, the gravity of the matter is not strong enough to reverse the expansion and so the Universe will expand forever.

It is straightforward to solve equation (4). If the Universe has the critical density, $\rho = \rho_c$, then the constant $C = 0$ and so,

$$\dot{R} = \frac{A}{R^{1/2}},$$

i.e.

$$R^{1/2} dR \propto dt,$$

and so integrating gives the solution,

$$R \propto t^{2/3}, \qquad \text{if } \Omega = 1.$$

The solutions for open and closed universes can be written in terms of a *development angle* θ,

$$\begin{aligned} R &= a(\cosh\theta - 1), \qquad t = b(\sinh\theta - \theta) \\ a &= \frac{\Omega_0}{2(1-\Omega_0)}, \qquad b = \frac{\Omega_0}{2H_0(1-\Omega_0)^{3/2}}, \qquad \text{if } \Omega_0 < 1, \end{aligned}$$

and

$$\begin{aligned} R &= a(1-\cos\theta), \qquad t = b(\theta - \sin\theta) \\ a &= \frac{\Omega_0}{2(\Omega_0 - 1)}, \qquad b = \frac{\Omega_0}{2H_0(\Omega_0-1)^{3/2}}, \qquad \text{if } \Omega_0 > 1. \end{aligned}$$

You can verify the solutions for the open and closed universes by writing the derivative in equation (4) in terms of the development angle θ rather than time t.

21.2 The cosmological redshift

The solutions of the Friedmann equations give us a time varying scale factor $R(t)$, so these models are dynamical. The Universe is evolving – the mean density, for example, declines as $1/R^3$ as the scale factor increases. We can see directly that these model universes are expanding by analysing the redshift of light signals. Let's begin with the FRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-Kr^2)} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right], \tag{8}$$

and suppose that we have two observers A abd B separated by a coordinate distance r_{AB}.

Observer A emits a light pulse at time t_e which reaches B at time t_0. Since $ds = 0$ along the path of a photon, the metric (8) requires

$$\int_{t_e}^{t_0} \frac{dt}{R(t)} = \int_0^{r_{AB}} \frac{dr}{(1-Kr^2)^{1/2}}.$$

Now let's suppose that A emits a second pulse at time $t_e + \delta t_e$ which arrives at B at time $t_0 + \delta t_0$, then again from the metric (8),

$$\int_{t_e+\delta t_e}^{t_0+\delta t_0} \frac{dt}{R(t)} = \int_0^{r_{AB}} \frac{dr}{(1-Kr^2)^{1/2}} = \int_{t_e}^{t_0} \frac{dt}{R(t)}$$

i.e.,

$$\int_{t_0}^{t_0+\delta t_0} \frac{dt}{R(t)} = \int_{t_e}^{t_e+\delta t_e} \frac{dt}{R(t)},$$

and so,

$$\frac{\delta t_0}{R(t_0)} = \frac{\delta t_e}{R(t_e)}.$$

As a consequence of this result, light of frequency ν_e at A will have a different frequency, ν_0, by the time it reaches observer B,

$$\frac{\nu_0}{\nu_e} = \frac{\delta t_e}{\delta t_0} = \frac{R(t_e)}{R(t_0)}.$$

The *redshift* z is defined as

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{\nu_e}{\nu_0} - 1,$$

and so,

$$1 + z = \frac{R(t_0)}{R(t_e)}. \tag{9}$$

If the Universe is expanding, $R(t_0) > R(t_e)$, and so this equation tells us light received from a distant source is *redshifted*[1].

If the source of the radiation is nearby we can write $t_e \approx t_0 - \delta t$, where $\delta t \ll t_0$, and expand the denominator in (9) in a Taylor series,

$$1 + z = \frac{R(t_0)}{R(t_0 - \delta t)} \simeq \frac{R(t_0)}{R(t_0) - \dot{R}_0 \delta t + \ldots} \approx 1 + \frac{\dot{R}_0}{R_0}\delta t + \ldots \tag{10}$$

So if, for example, we measure the spectrum of a galaxy, we can define a *recession velocity*, $v = cz$, from the redshift of the spectral lines. Equation (10) then tells us that the recession velocity will be given by

$$v = \left(\frac{\dot{R}_0}{R_0}\right) c\delta t = \underbrace{H_0}_{\downarrow \atop \text{Hubble "constant"}} d, \tag{11}$$

i.e. galaxies will appear to recede from us with a recession speed that is proportional to their distance from us. This, of course, is Hubble's law and the proportionality 'constant' H_0 is called the Hubble constant. Hubble discovered the expansion of the Universe in 1929, by comparing redshifts with distance measurements to nearby galaxies (derived from the period-luminosity relation of Cepheid variables). His results suggested a linear recession law, as in equation (11). The Universe around us is expanding, as predicted by the simple cosmological models of General Relativity. This is an amazing result. It implies that the Universe started off at high density at some finite time in the past. You will notice from equation (11) that the Hubble "constant" has the dimensions of inverse time. As we will see in Section 21.5, the quantity $1/H_0$ gives the age of the Universe to within a factor of order unity. From equation (10) it is clear that the Hubble parameter is not a constant of nature but varies with time. In general, $H = \dot{R}/R$ and since $R(t)$ varies with time so does the Hubble "constant". H_0 is the value of the Hubble parameter *measured at the present time* t_0.

21.3 The density parameter and dark matter

As we have seen, the curvature and future history of these simple cosmological models depends on whether the mean density of matter in the Universe is less than or greater than the critical density

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$

Modern measurements of the Hubble constant (based on Cepheid and Type Ia supernovae distances measured with the Hubble Space Telescope) indicate that value of the Hubble constant is

$$H_0 = 65 \pm 10 \text{ km/sec/Mpc}. \tag{12}$$

[1] If the universe were collapsing, the light received would be blueshifted.

(Note that 1 Mpc $= 3.086 \times 10^{24}$cm). The Hubble constant is extremely difficult to measure. In fact, Hubble's original estimate was nearly 10 times higher than the modern value. The experimental situation has changed considerably since the launch of the Hubble Space Telescope but systematic errors dominate the error budget in (12) and so the Hubble constant remains a relatively uncertain number. Because of this, in textbooks and research articles you will often see the Hubble constant parameterised as

$$H_0 = 100h \text{ km/sec/Mpc}.$$

If the true value of the Hubble constant is 65 km/sec/Mpc, then the parameter h is 0.65. This parameterization allows one to keep track of the Hubble constant in equations so that it is easy to see how quantities are affected by uncertainties in its value. For example, if we evaluate the expression for the critical density, we find

$$\rho_c = 1.88 \times 10^{-29} h^2 \text{ g/cm}^3.$$

Inserting $h = 0.65$, the critical density is

$$\begin{aligned} \rho_c &= 7.9 \times 10^{-30} \text{ g/cm}^3, \\ &= 4.6 \text{ protons/m}^3. \end{aligned}$$

The critical density turns out to be a few[2] protons per cubic metre. As you will appreciate, this is an exceedingly low density. Nevertheless, if the mean density in our Universe exceeds a few protons per cubic metre, then the gravity of the matter will be strong enough to eventually reverse the expansion of the Universe.

What is the mean density of the Universe? It is straightfoward to estimate the contribution of ordinary (hydrogen-burning) stars to the mean density. In units of the critical density, ordinary stars contribute

$$\Omega_* \approx 0.008.$$

This is nowhere near the critical value $\Omega = 1$. If ordinary stars were the only type of matter in the Universe, then the mean density would be less than a percent of the critical density. However, dynamical measurements indicate a much higher matter density of $\Omega \approx 0.3$. This is still short of the critical density, but much higher than the mean density of ordinary stars. The dynamical evidence suggests that most of the matter in the Universe is dark, invisible and very different to ordinary stars. The favoured candidate for this *dark matter* is some form of stable weakly interacting supersymmetric particle such as a gravitino (the supersymmetric partner of the graviton). Searches are underway to find such particles by detecting their interactions with ordinary matter.

21.4 Einstein's static universe

The models that we have constructed from the field equation are *evolving* cosmologies. We now know, of course, that the Universe is expanding and so there is no conflict with the field equations. But it is interesting historically to look at Einstein's static model of the

[2]The fact that this is a number of order unity is an accident!

Universe. Einstein derived his field equations well before the discovery of the expansion of the Universe and he was worried that he could not find static cosmological solutions. He therefore introduced the cosmological constant with the sole purpose of constructing static solutions. You can see how this works from the Friedmann equations. I will assume that the pressure of matter can be ignored, in which case the Friedmann equations including a non-zero cosmological constant are

$$3\frac{\ddot{R}}{R} = -4\pi G\rho + \Lambda, \tag{13a}$$

and

$$\frac{\dot{R}^2}{R^2} + \frac{K}{R^2} = \frac{8}{3}\pi G\rho + \frac{1}{3}\Lambda \tag{13b}$$

If $\Lambda > 0$, then we can find a solution in which the Universe is static, $\ddot{R} = \dot{R} = 0$. Equations (13) then give

$$\left.\begin{array}{rcl} \Lambda & = & 4\pi G\rho \\ K/R^2 & = & \Lambda \end{array}\right\}$$

Note the following points about this solution:

- A non-zero value of Λ is required for a static universe.
- The Universe is necessarily closed ($K > 0$).
- The cosmological constant must be fine tuned to match the density of the universe.

The last of these points is the most worrying. To construct a static model of the Universe, a constant of nature (Λ) is equated to the density of matter in the Universe. If we add or subtract one proton from this Universe (or convert some matter into radiation), we will disturb the finely-tuned balance between gravity and the cosmological constant and the Universe will begin to expand or contract. Einstein's static universe is *unstable* and is theoretically very unattractive.

21.5 The age of the Universe

We can rewrite the Friedmann equation,

$$\dot{R}^2 = \frac{8\pi G}{3}\rho_m R^2 + \frac{\Lambda}{3}R^2 + K,$$

as,

$$\frac{\dot{R}^2}{R^2} = H_0^2\left[\Omega_m\frac{R_0^3}{R^3} + \frac{\Lambda}{3H_0^2} + \frac{K}{R^2H_0^2}\right], \tag{14}$$

where $\Omega_m = \rho_m/\rho_c$ is the density parameter of the matter. Now define two new density parameters,

$$\begin{aligned} \Omega_\Lambda &= \frac{\Lambda}{3H_0^2}, \\ \Omega_K &= 1 - \Omega_m - \Omega_\Lambda = \frac{K}{R_0^2H_0^2}, \end{aligned}$$

then we can write equation (14) as

$$\frac{\dot{R}^2}{R^2} = H_0^2 \left(\Omega_m \frac{R_0^3}{R^3} + \Omega_\Lambda + \Omega_K \frac{R_0^2}{R^2} \right). \tag{15}$$

We can integrate (15) to give

$$t_0 = \frac{1}{H_0} \int_0^{R_0} \frac{dR/R}{\left[\Omega_m R_0^3/R^3 + \Omega_\Lambda + \Omega_K R_0^2/R^2\right]^{1/2}}. \tag{16}$$

The integral is dimensionless, so we can write

$$t_0 = \frac{1}{H_0} f(\Omega_m \ , \ \Omega_\Lambda)$$

where f is a number of order unity. The age of the Universe is therefore the inverse of the Hubble constant

$$\frac{1}{H_0} = 9.79 \times 10^9 h^{-1} \mathrm{yrs}$$

times a number of order unity. For an Einstein-de Sitter universe ($\Omega_m = 1$, $\Omega_\Lambda = 0$) we can do the integral analytically to show that $f = 2/3$. However, for general values of Ω_m and Ω_Λ we cannot express the integral (16) in terms of elementary functions and so we have to resort to numerical integration. In the following table I have numerically integrated equation (16) for several values of Ω_m and Ω_Λ:

Age of the Universe in Gyr				
		H_0 in km/sec/Mpc		
Ω_m	Ω_Λ	60	65	90
1.0	0.0	13.1	10.0	7.2
0.3	0.0	15.8	12.2	8.8
0.3	0.7	18.9	14.5	10.5

Estimates of the ages of the oldest stars in globular clusters suggest an age of

$$t_{\mathrm{stars}} \approx 11.5 \pm (1.3) \ \mathrm{Gyr},$$

where the uncertainty is dominated by uncertainties in the theory of stellar evolution. If we lived in an Einstein-de Sitter universe and the Hubble constant is really $H_0 = 65$ km/sec/Mpc, then the oldest stars would be older than the Universe! This is not, therefore, a viable cosmology. If, however, $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$ as suggested by recent observations of Type Ia supernovae and the cosmic microwave background radiation, then a universe with $H_0 = 65$ km/sec/Mpc is comfortably older than the oldest stars. This is an additional argument in favour of a non-zero value for the cosmological constant. But beware of the large experimental uncertainties – I would not take this particular argument in favour of a cosmological constant too seriously.

21.6 Distances in FRW cosmologies

From the FRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{(1-Kr^2)} + r^2 d\Omega^2\right],$$

we can define a number of different measures of distance. The distance r is a comoving coordinate which I have referred to as the *coordinate distance.* Distances measures in an expanding universe can sometimes be confusing. Let us, for example, consider the distance to a distant galaxy. The light that is received from the galaxy was emitted when the Universe was younger than it is now because light travels at a finite speed c. Evidently, as we look at more distant objects, we see them as they were at an earlier time in the Universe's history *when proper distances were smaller* (since the Universe is expanding). What, therefore, do we mean by the 'distance' to a galaxy? In fact interpreting and calculating distances in an expanding Universe is easy, but you must be clear about what you mean by 'distance'. I will illustrate what I mean by looking at two important distance measures, *luminosity distance* and *angular diameter distance.* These distance measures form the basis for observational tests of the geometry of the Universe. They will also teach you how to interpret the FRW metric.

21.6.1 Luminosity distance

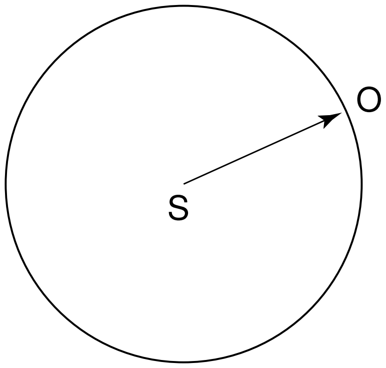

Figure 2:

Consider a source S that is separated by a coordinate distance r from an observer O as shown in Figure 2. Assume that the source S emits photons that are detected by the observer O at the present time t_0. Clearly, the photons were emitted by the source S at some earlier time t_e, since it takes light a finite time to travel from S to O.

Let's assume that S emits N photons isotropically and uniformly over a time interval Δt_e. From the FRW metric, we can see that the radiation will be distributed isotropically over an area,

$$4\pi R^2(t) r^2,$$

(the angular part of the FRW metric is the same as that of a three dimensional sphere of radius $R(t)r$ in Euclidean space) and so the number of photons collected per unit area by an observer O is

$$\Delta N = \frac{N}{4\pi R^2(t_0) r^2}.$$

But, each photon received by O is redshifted in frequency by

$$\nu_0 = \frac{\nu_e}{(1+z)},$$

and the time interval over which they are received by O is longer,

$$\Delta t_0 = \Delta t_e (1+z),$$

and so the observed *flux* at O is

$$F = \frac{h\nu_0 \Delta N}{\Delta t_0} = \frac{h\nu_e N}{\Delta t_e 4\pi R^2(t_0) r^2 (1+z)^2},$$

i.e.,

$$F = \frac{\mathrm{L}_s}{4\pi R_0^2 r^2 (1+z)^2}$$

where L_s is the luminosity of the source.

We can write this equation as

$$F = \frac{\mathrm{L}_s}{4\pi d_L^2}$$

which is just what we would write for Euclidean space for a source of luminosity L_s and Euclidean distance d_{L}. This equation *defines the luminosity distance*, d_{L},

$$d_{\mathrm{L}} = R_0 r (1+z). \tag{17}$$

21.6.2 Angular diameter distance

Imagine a rod of proper length ℓ at distance S from an observer at postion O. Let us assume that the rod is perpendicular to the line–of–sight as in Figure 3. Then the angle subtended by the rod at O can be read directly from the metric

$$\ell = R(t_e) r \gamma,$$

which we can write in terms of the scale factor at the present time as

$$\ell = \frac{R_0 r \gamma}{(1+z)}.$$

We can therefore define an *angular diameter* distance, d_A,

$$\ell = d_A \gamma,$$

which is just what we would write for Euclidean space, where the angular diameter distance is related to the coordinate distance r by

$$d_A = \frac{R_0 r}{(1+z)}. \tag{18}$$

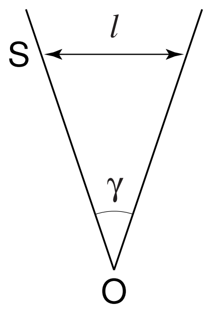

Figure 3:

Notice that the angular diameter and luminosity distances of equations (17) and (18) are related, to each other by

$$\frac{d_L}{d_A} = (1+z)^2\,,$$

which depends only on the redshift of the source S.

21.6.3 Relationship of coordinate distance to redshift

All that remains to get useful expressions for the luminosity and angular diameter distance is to relate the coordinate distance r of an object to its redshift.

From the FRW metric, and applying the condition $ds = 0$, the coordinate distance r along the path of a photon is

$$\int_0^r \frac{dr'}{(1-Kr^2)^{1/2}} = \int_t^{t_0} \frac{dt}{R}. \tag{19}$$

Using the Friedmann relation (for $\Lambda = 0$),

$$\dot{R}^2 + K = \frac{8\pi G}{3}\rho R^2,$$

we can write integral on the right hand side of (19) as

$$\int_0^r \frac{dr'}{(1-Kr^2)^{1/2}} = \int_{R(t)}^{R(t_0)} \frac{dR}{\left[R_0^3 H_0^2 \Omega_0 R - KR^2\right]^{1/2}}. \tag{20}$$

These integrals are simple if we restrict to a spatially flat universe with $\Omega_0 = 1$ ($i.e.$ $K = 0$):

$$r = \int_{R(t)}^{R(t_0)} \frac{dR}{R^{3/2} H_0 R^{1/2}} = \left[\frac{2R^{1/2}}{R_0^{3/2} H_0}\right]_{R(t)}^{R_0},$$

and so inserting the factor c to get the right dimensions,

$$r = \frac{2c}{R_0 H_0}\left[1 - (1+z)^{-1/2}\right]. \tag{21}$$

This is the required relation between the coordinate distance r to a source detected with a redshift z. From this relation you can see that the luminosity distance in a $K = 0$, $\Lambda = 0$, cosmology is given by

$$d_L = \frac{2c}{H_0}(1+z)\left[1-(1+z)^{-1/2}\right], \tag{22}$$

and the angular diameter distance by,

$$d_A = \frac{2c}{H_0}\frac{1}{(1+z)}\left[1-(1+z)^{-1/2}\right]. \tag{23}$$

Figures 4 and 5 show the luminosity and angular diameter distances plotted against redshift for various values of Ω_0.

Notice that equation (23) has a minimum at a redshift $z = 5/4$. At small redshifts ($z \ll 5/4$), the angular diameter of a source of fixed proper length declines linearly with redshift ($d_A = cz/H_0$) just as we would expect in a Euclidean universe. But at redshifts $z > 5/4$, the angular size of a source of standard length would actually *increase* with redshift. A very high redshift galaxy (if such a thing existed) would cast a large, but dim, ghostly image on the sky. The physical reason for this is that the light from a distant object was emitted when the Universe was much younger than it is now – the object was close to us when the light was emitted and so looks big!

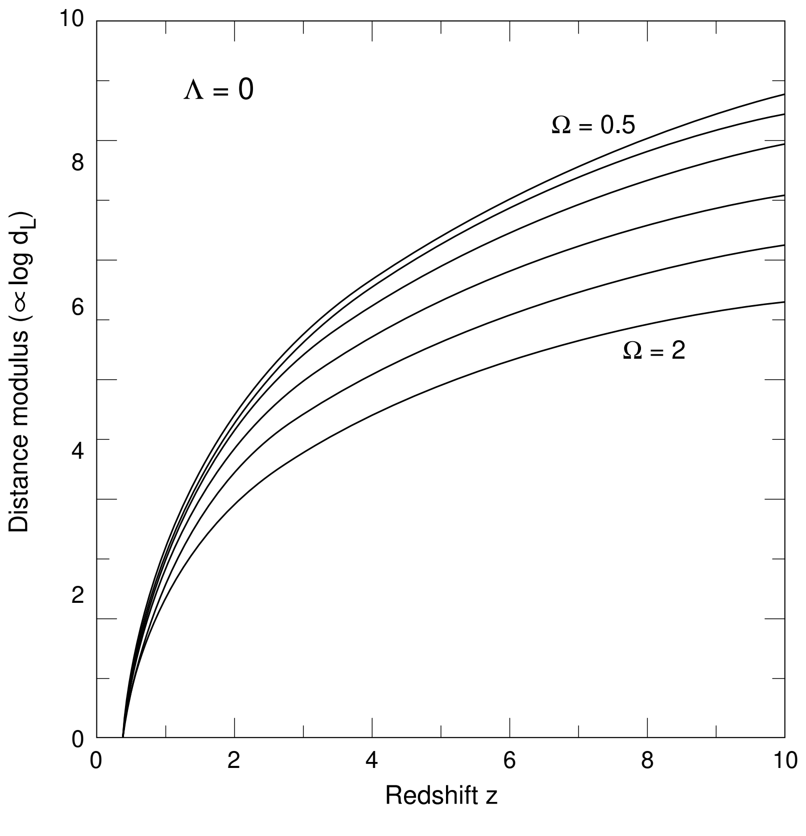

Figure 4: The luminosity distance as a function of redshift for FRW models with various values of the density parameter Ω_0. (The cosmological constant is assumed to be zero.)

The relations between redshift and luminosity, or angular diameter, distance form the basis of observational tests of the geometry of the Universe. All one needs is a standard candle

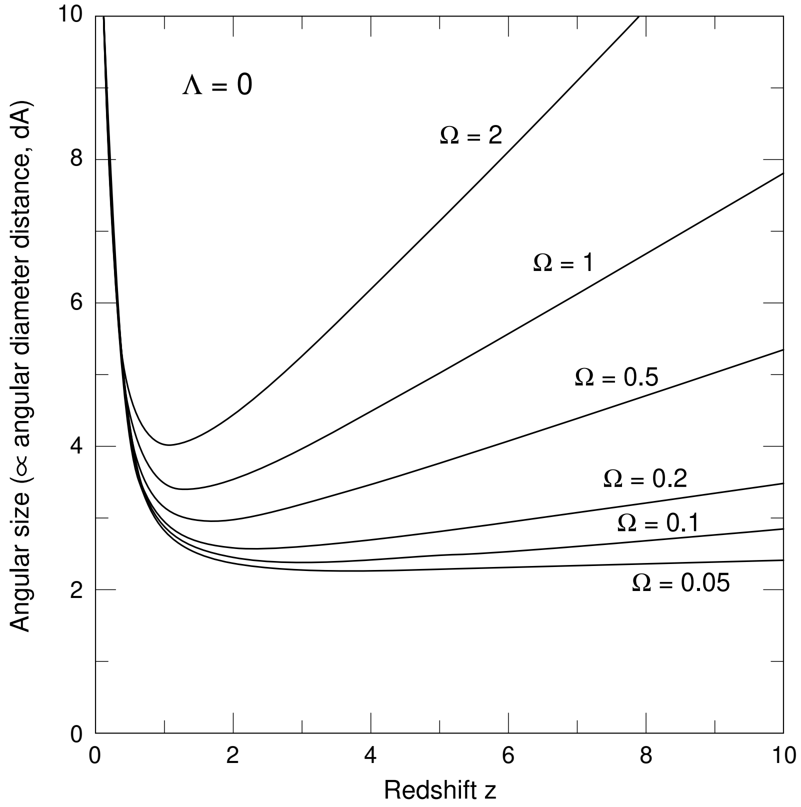

Figure 5: The angular distance as a function of redshift for FRW models with various values of the density parameter Ω_0. (The cosmological constant is assumed to be zero.)

(for the application of the luminosity distance-redshift relation) or a standard ruler (for the application of angular diameter distance -redshift relation). Comparison with the relations predicted in Figures 4 and 5 can then fix the values of Ω_m, Ω_Λ and the curvature of the Universe. Unfortunately, standard candles and standard rulers are hard to find in the Universe! Nevertheless, in the last three years there has been remarkable progress, using distant Type Ia supernovae as standard candles, and anisotropies of the cosmic microwave background radiation as a standard ruler. The results of these observations suggest that we live in a spatially flat Universe with $\Omega_m \approx 0.3$ and $\Omega_\Lambda = 0.7$.